Unified Model for Collector Charge in Heterojunction Bipolar Transistors

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***Abstract*—The base–collector capacitance and the collector transit-time in GaAs-based heterojunction bipolar transistors depend not only on base–collector voltage, but also on collector current. This has to be taken into account in a large-signal model. However, since collector transit-time and capacitance are both caused by the charge stored in the collector space–charge region, it is not possible to model them independently of each other. This paper investigates the interrelation between collector capacitance and transit-time due to transcapacitance effects, and presents an analytical unified description for both quantities, that is derived from measurement-extracted small-signal equivalent circuits. The model is verified by comparison of simulation and measurement data.**

***Index Terms*—Equivalent circuit, heterojunction bipolar transistor, semiconductor device modeling.**

I. Introduction

THE collector of heterojunction bipolar transistors (HBTs) is commonly weakly doped in order to reduce base–collector capacitance and to increase break-down voltage. The drawback is that high-current injection occurs already at low current densities. As a consequence, base–collector capacitance and collector transit-time are modulated [1]. This influences the HBTs RF behavior and especially has a negative impact on linearity [2]. Therefore, it has to be taken into account in large-signal modeling. Since physically, both capacitance and transit-time are governed by the charge stored in the collector, definition of a current-dependent collector capacitance will inherently lead to variations in transit-time [3].

While the current dependence of the base–collector capacitance is accounted for in several recently published large-signal models [1], [2], [4], [5], the interdependence of capacitance and transit-time has implicitly been mentioned, but not been discussed in detail yet. Traditional nonquasi-static models for HBTs, on the other hand, account for the base transit-time by introducing a base charge in a similar way [6], [7], but neglect the current dependence of the collector capacitance.

In this paper, first the influence of collector charge on transit-time is investigated. Then, a unified model for base–collector capacitance and collector transit-time is presented. A single formula for collector charge Q_c is given that describes both quantities. It is derived empirically from small-signal parameters of GaAs-based HBTs and is implemented into a compact large-signal model.

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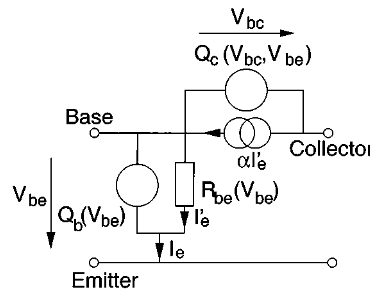

Fig. 1. Large-signal equivalent circuit of intrinsic HBT.

II. Bias Dependent Collector Charge

By definition, a capacitance is given as the derivative of charge with respect to voltage. In case of the HBT's base–collector capacitance $C_{\rm bc}$, the charge Q_c is the total charge stored in the base–collector space–charge region, which in case of a npn-transistor is given by the charge of the dopants, and the charge of the electrons due to collector current. Hence, $C_{\rm bc}$ becomes a function of collector current I_c and base–collector voltage $V_{\rm bc}$ as well, in contrast to the capacitance of a single pn-junction that depends only on the voltage across it. Since I_c is a function of $V_{\rm bc}$ and base–emitter voltage $V_{\rm be}$, Q_c for simplicity is defined as a function of both voltages: $Q_c = f(V_{\rm be}, V_{\rm bc})$. Fig. 1 shows the intrinsic part of the HBT large-signal equivalent circuit. It consists of the charges of base–emitter and base–collector p-n-junctions, the nonlinear resistance of the base–emitter p-n-junction, and the current source α. For simplicity, the high resistance of the reverse biased base–collector junction is not shown, nor the extrinsic base–collector diode and the parasitics.

In order to obtain the corresponding small-signal equivalent circuit, all of its elements are to be derived with respect to the voltages. Q_c depends on two voltages and, therefore, yields $C_{\rm bc}$ and a transcapacitance $C_{\rm tr}$ as well

$$C_{\rm bc} = \frac{\partial Q_c}{\partial V_{\rm bc}} = f(V_{\rm be}, V_{\rm bc}) \tag{1}$$

$$C_{\rm tr} = \frac{\partial Q_c}{\partial V_{\rm be}} = f(V_{\rm be}, V_{\rm bc}). \tag{2}$$

As shown in Fig. 2, $C_{\rm tr}$ may be understood as a voltage dependent current source that is multiplied with $j\omega$.

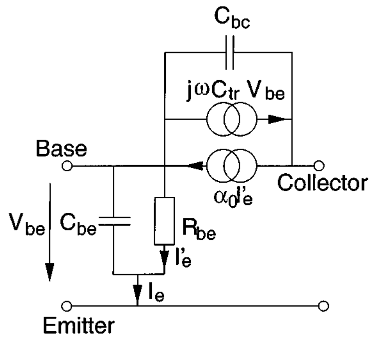

Fig. 2. Linearized large-signal (i.e., small-signal) equivalent circuit of intrinsic HBT.

It is worth mentioning that $C_{\rm tr}$ is of physical origin, and not only a mathematical problem. It reflects the fact that I_c is mainly controlled by $V_{\rm be}$. It may be understood as a collector transit-time.

Since $C_{\rm tr}$ lies in parallel with the current gain α, both can be joined to form a total current gain α' in the small-signal equivalent circuit. In order to do so, both current sources in Fig. 2 are rewritten as follows:

$$j\omega C_{\rm tr} V_{\rm be} \Leftrightarrow j\omega C_{\rm tr} \frac{1}{Y_{\rm be}} I_e \tag{3}$$

$$\alpha_0 I'_e \Leftrightarrow \alpha_0 \frac{G_{\rm be}}{Y_{\rm be}} I_e \tag{4}$$

$$\rightsquigarrow \ \alpha' I_e = \frac{\alpha_0 - j\omega C_{\rm tr}/G_{\rm be}}{1 + j\omega/\omega_\alpha} I_e \tag{5}$$

with $G_{\rm be} = 1/R_{\rm be}$, $Y_{\rm be} = G_{\rm be} + j\omega C_{\rm be}$ and $\omega_\alpha = 1/(R_{\rm be} C_{\rm be})$.

The locus of α' in the complex plain is a half-circle, starting at α_0 for $\omega = 0$, approaching $-C_{\rm tr}\omega_\alpha/G_{\rm be}$ for $\omega \rightarrow \infty$. The transcapacitance therefore does not yield excessive gain at highest frequencies, but reaches a constant value. The total gain of the base–collector branch then is additionally limited by $j\omega C_{\rm bc}$.

Equation (5) is approximated in order to estimate the total transit-time τ

$$\alpha' \approx \frac{\alpha_0}{1 + j\omega(1/\omega_\alpha + C_{\rm tr}/G_{\rm be})} = \frac{\alpha_0}{1 + j\omega\tau}. \tag{6}$$

In the small-signal parameter extraction, only α' is visible, and only τ can be extracted reliably.

Three conclusions are to be drawn from this section.

- The current dependence of $C_{\rm bc}$ leads to a transcapacitance.
- The transcapacitance adds a value $C_{\rm tr}/G_{\rm be} = \tau_c$ to the total transit-time τ. Therefore, it can be understood as the origin of the collector transit-time. This agrees with the finding [1] that both τ_c and $C_{\rm bc}$ can be assumed to be independent of temperature, in contrast to ω_α.
- The formula for Q_c used in a large-signal model has to represent $C_{\rm bc}$ as well as τ_c. A unified collector-charge model is required.

III. Implementation Into Large-Signal Model

The capacitance of the space-charge region of reverse-biased pn-junctions is given by

$$Q'_c = -C_{jc0}\phi \frac{(1 - V_{\rm bc}/\phi)^{1-m}}{1-m} \tag{7}$$

$$C'_{\rm bc} = C_{jc0}(1 - V_{\rm bc}/\phi)^{-m}. \tag{8}$$

The base–collector space-charge region of a real HBT, however, is limited by the collector width, therefore $\lim_{V_{\rm bc}\rightarrow -\infty} C_{\rm bc} = C_{\rm min}$. In our approach, for $I_c = 0$, this is modeled by

$$Q_c = Q'_c + C_{\rm min} V_{\rm bc} \tag{9}$$

$$C_{\rm bc} = C'_{\rm bc} + C_{\rm min}. \tag{10}$$

The advantage of this approach is that one has a single continuous description for the complete voltage range.

The electrons of the collector current first lower the effective collector doping, leading to a reduction of $C_{\rm bc}$. While the current dependence may be linearly approximated [1], [2], [5], this leads to a τ_c that is constant up to the current when $C_{\rm bc}$ reaches its minimum value, and then drops to zero. The following empirical expression on the other hand, is well-behaved:

$$Q_{c,\,{\rm med}} = \left(1 - \frac{2\, I_c/I_0}{1 + (I_c/I_0)^2}\right) Q'_c + C_{\rm min} V_{\rm bc} \tag{11}$$

$$C_{bc,\,{\rm med}} = \left(1 - \frac{2\, I_c/I_0}{1 + (I_c/I_0)^2}\right) C'_{\rm bc} + C_{\rm min} \tag{12}$$

$$C_{tr,\,{\rm med}} = 2\,\frac{\partial I_c}{\partial V_{\rm be}}\,\frac{I_0 I_c^2 - I_0^3}{(I_0^2 + I_c^2)^2}\, Q'_c \tag{13}$$

for $I_c < I_0$, with the fitting parameter I_0. For $I_c > I_0$, $Q_{c,\,{\rm med}} = C_{\rm min} V_{\rm bc}$, $C_{bc,\,{\rm med}} = C_{\rm min}$, $C_{tr,\,{\rm med}} = 0$. $I_c \approx \alpha I_e \rightsquigarrow \partial I_c/\partial V_{\rm be} \approx I_c/(n_e V_{\rm th})$ with the base–emitter ideality factor n_e, and the thermal voltage $V_{\rm th}$. The index "med" indicates that this formulas are valid in the range of medium currents. They approximate the linear behavior of $C_{\rm bc}$ at low currents, but at $I_c = I_0$, when $C_{\rm bc}$ reaches its minimum value $C_{\rm min}$, $C_{\rm tr}$ becomes zero. Thereby, regular expressions are assured which is not the case for a purely linear approximation of $C_{\rm bc}$.

The onset of base push-out, or Kirk effect, depends on I_c and on $V_{\rm bc}$ as well. For simplicity, a constant current I_k is defined that corresponds to the condition that the electrons of the collector current fully compensate the collector doping [1], [2]. Above I_k, increasing current reduces the base–collector space-charge region and, therefore, $C_{\rm bc}$ increases again. This behavior can be approximated by

$$Q_{c,\,{\rm hi}} = \left(\frac{I_c}{I_k} - 1\right)^2 Q''_c \tag{14}$$

$$C_{bc,\,{\rm hi}} = \left(\frac{I_c}{I_k} - 1\right)^2 C''_c \tag{15}$$

$$C_{tr,\,{\rm hi}} = \frac{\partial I_c}{\partial V_{\rm be}}\,\frac{2}{I_k}\left(\frac{I_c}{I_k} - 1\right) Q''_c \tag{16}$$

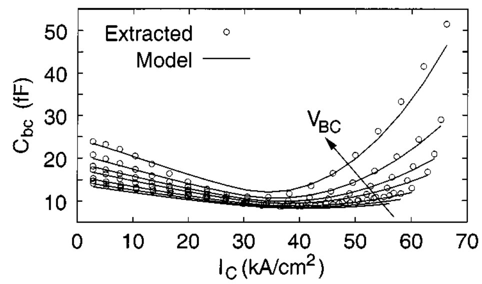

Fig. 3. Extracted (○) and modeled (–) intrinsic base–collector capacitance $C_{\rm bc}$. Parameter is $V_{\rm BC} = -4.5, \ldots, 0$ V.

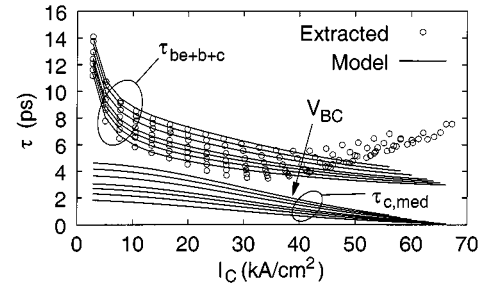

Fig. 4. Extracted (○) and modeled (—) transit-times. The contribution of eq. (13) to the transit-time $\tau_{c,\,\rm med}$, and the total transit-time for the lower current range τ_{be+b+c}, that consists of base–emitter charging time, temperature dependent base and collector transit-times. Parameter is $V_{\rm BC} = -4.5, \ldots, 0$ V.

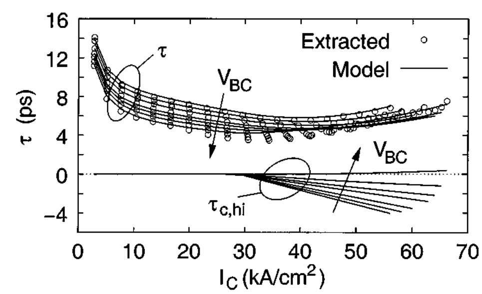

Fig. 5. Extracted (○) and modeled (—) transit-times. The contribution of (16) to the transit-time $\tau_{c,\,\rm hi}$, and the total transit-time τ. Parameter is $V_{\rm BC} = -4.5, \ldots, 0$ V.

for $I_c > I_k$. For $I_c < I_k$: $Q_{c,\,\rm hi} = C_{bc,\,\rm hi} = C_{tr,\,\rm hi} = 0$. C_c'' stands for a capacitance formula as in (8) and (12), but with modified parameter m. While in p-n-junctions, m is restricted to a range of $1/3, \ldots, 0.5$, it is used in this formula to compensate for the neglect of the voltage dependence of I_k. In case of the HBTs under investigation, $m = 1$. The index "hi" indicates that these formulas model excess charge at high currents. $Q_c'' = \int C_c''\, dV_{\rm bc}$.

The full formula for the collector charge is now given by

$$Q_c = Q_{c,\,\rm med} + Q_{c,\,\rm hi}. \tag{17}$$

In addition of τ_c, that has been studied in detail so far, base transit-time and emitter charging-time are modeled by the time constant $1/\omega_\alpha$[see (6)] of the base–emitter junction. While the emitter charging-time is given by the base–emitter p-n-junctions depletion capacitance, the diffusion capacitance, and base transit-time is modeled by

$$Q_b' = (\tau_f + \tau_t \Delta T) I_c \tag{18}$$

$$Q_{b,\,\rm hi} = (\tau_k + \tau_{k,\,t}\Delta T) V_{\rm th} n_e \left(\frac{I_c}{I_k} - 1\right)^2 \tag{19}$$

where $Q_{b,\,\rm hi}$ describes the excess transit-time due to base push-out for $I_c > I_k$. τ_f describes the base transit-time and base–emitter diffusion capacitance, τ_t describes its temperature dependence, ΔT is the difference in temperature due to self-heating. The factor τ_k describes the slope of τ due to base push-out, and $\tau_{k,t}$ its temperature dependence.

IV. Results

The model has been developed for the FBH 4″ GaInP/GaAs HBT process [8]. It is implemented into a commercial circuit simulator and now in routine use for MMIC design [9]. Convergence of large-signal harmonic-balance simulations generally is good, since the new formula for Q_c and its derivatives are continuous and well-behaved functions. In order to obtain the collector-charge model parameters, first, the small-signal equivalent-circuit elements are determined using an analytical algorithm [10]. Then, the formulas given above are fitted to these values.

Measured and modeled $C_{\rm bc}$ of HBTs with an emitter size of $3 \times 15\,\mu$m^2 is shown in Fig. 3. Up to 35 kA/cm^2, $C_{\rm bc}$ is given by (12). Up to this current, the almost linear current dependence is still well approximated. The contribution of $C_{\rm tr}$ on τ is shown in Fig. 4. In the current range below the onset of Kirk-effect, mainly τ_c, τ_f, and the base–emitter charging time $\tau_{\rm be}$ contribute to τ. $\tau_{\rm be}$ exhibits a $1/I_C$ behavior, and τ_f increases linearly with temperature [1]. The collector transit-time τ_c, that is mainly responsible for the bias dependence of τ, is well approximated by (13).

In the example, Fig. 3, the Kirk effect sets in around $I_C = 35$ kA/cm^2. Although I_k is approximated as a constant value, $C_{\rm bc}$ is modeled well in a broad bias range by (15). Fig. 5 shows the contribution of (16) to τ_c. The higher the collector current, the narrower the space-charge region becomes, and as a consequence, the number of stored electrons is reduced. Furthermore, the negative value of the transcapacitance corresponds to $C_{\rm bc}$ that increases with current. Under base push-out condition, however, the total transit-time rapidly increases due to the effective base widening. Therefore, negative τ values are not observed nor modeled. Fig. 5 also provides data on the total transit-time τ.

Figs. 6 and 7 show a comparison of measured and simulated S-parameters at $I_C = 22$ kA/cm^2, $V_{CE} = 1.5$ V, 3 V, 6 V of HBTs with an emitter size of $3{\times}30\ \mu$m^2. At this current, transit frequency f_t is maximum for this sample, and, consequently, Kirk effect sets in at higher currents. The S-parameters are modeled well in case of the new model (see Fig. 6). The accuracy of S_{22} deteriorates significantly, if $C_{\rm bc}$ is assumed current independent (see Fig. 7). In both cases, the S-parameters are modeled with a large-signal model using the same set of parameters. Only C_{jc0} of (8) and τ_f of (18) are adjusted in the simplified case in order to obtain the best fit for S-parameters and $|H_{21}|^2$ at the bias point $I_C = 22$ kA/cm^2, $V_{CE} = 3$ V. Consequently, C_{jc0} in this case is lower than the value one determines by measuring the base–collector p-n-junction.

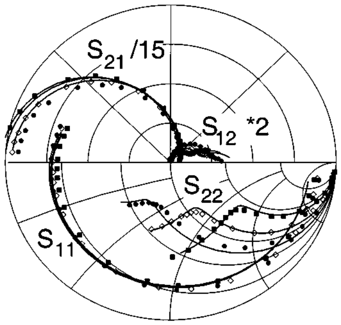

Fig. 6. S-parameters of 3×30 μm^2 HBT, measured (symbols) and modeled with new large-signal model. $I_C = 22$ kA/cm^2, $V_{CE} = 1.5$ V (●), 3 V (◇), 6 V (■), $f = 150$ MHz–40 GHz.

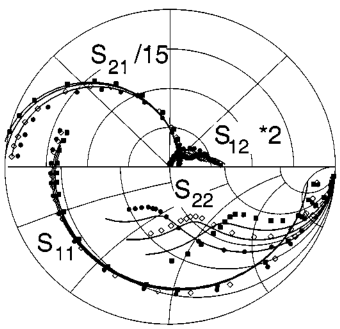

Fig. 7. S-parameters of 3×30 μm^2 HBT, measured (symbols) and modeled with large-signal model neglecting current dependence of $C_{\rm bc}$. $I_C = 22$ kA/cm^2, $V_{CE} = 1.5$ V (●), 3 V (◇), 6 V (■), $f = 150$ MHz–40 GHz.

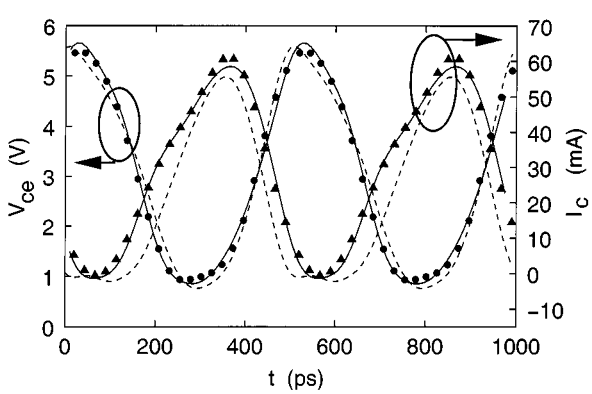

Fig. 8. Current and voltage waveforms of 3×30 μm^2 HBT, measured (symbols) and modeled with new model of collector charge (solid line) and neglecting current dependence of $C_{\rm bc}$ and τ (dashed line). $I_C = 33$ kA/cm^2, $V_{CE} = 3$ V, $f = 2$ GHz.

Large-signal waveform measurements for the same device are shown in Fig. 8. The current sweep reaches into the range of base push-out. Again, the improvement in accuracy using the new model is visible. Even at this relatively low frequency ($f \approx 0.06 f_t$), the current dependence of $C_{\rm bc}$ and τ plays an important role.

V. Conclusion

High-current injection is observed in the collector of state-of-the-art GaAs-based HBTs. It modulates the base–collector capacitance $C_{\rm bc}$ as well as the collector transit-time τ_c. This behavior has to be accounted for in large-signal models, since it has an impact on small- and large-signal RF behavior and on distortion characteristics.

It is shown in this work that $C_{\rm bc}$ and τ_c can not be modeled independently of each other in a large-signal model. Furthermore, the current-dependence of $C_{\rm bc}$ inherently leads to a transcapacitance. This transcapacitance can be understood as the origin of τ_c. Measurement-extracted values of both quantities are taken into account in order to derive an analytical model for a unified collector charge Q_c, that is capable of describing both $C_{\rm bc}$ and τ_c. The new formulas improve the accuracy of an HBT model in a broad range of bias points.

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